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# Topology and confinement at $T \neq 0$ : calorons with non-trivial holonomy<sup>1</sup>

P. Gerhold\*, E.-M. Ilgenfritz\*, M. Müller-Preussker\*, B.V. Martemyanov<sup>†</sup>  
and A.I. Veselov<sup>†</sup>

\*Humboldt-Universität zu Berlin, Institut für Physik, Newtonstr. 15, 12489 Berlin, Germany

<sup>†</sup>Institute for Theoretical and Experimental Physics, B. Cheremushkinskaya 25, Moscow 117259,  
Russia

**Abstract.** In this talk, relying on experience with various lattice filter techniques, we argue that the semiclassical structure of finite temperature gauge fields for  $T < T_c$  is dominated by calorons with non-trivial holonomy. By simulating a dilute gas of calorons with identical holonomy, superposed in the algebraic gauge, we are able to reproduce the confining properties below  $T_c$  up to distances  $r = O(4\text{fm}) \gg \rho$  (the caloron size). We compute Polyakov loop correlators as well as space-like Wilson loops for the fundamental and adjoint representation. The model parameters, including the holonomy, can be inferred from lattice results as functions of the temperature.

**Keywords:** Yang-Mills theory, lattice, caloron, non-trivial holonomy, confinement

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Instanton or caloron models of QCD successfully describe many non-perturbative features of hadron physics, in particular chiral symmetry breaking and the  $U_A(1)$  anomaly (for reviews see [1, 2]). However, they fail to describe confinement unless they are endowed with long-range correlations. This is the case for instantons in the regular gauge [3] and has been discussed also at this conference [4]. An attractive alternative, at least for non-zero temperature  $T$ , is based on new caloron solutions with non-trivial holonomy [5, 6, 7, 8], worked out in various aspects by Kraan and van Baal. At this symposium we were happy to listen to a review talk [9] by Pierre van Baal after his recovery.

The new caloron solutions - we call them KvBLL calorons - have characteristic properties which distinguish them from the old BPST instantons [10] or Harrington-Shepard (HS) calorons [11]. Like the latter, the new calorons are (anti)selfdual with integer topological charge and periodic in Euclidean time with the period  $1/T$ . The difference is a non-trivial asymptotic behaviour of  $A_4(x)$  such that the Polyakov loop

$$P(\vec{x}) = \hat{P} \exp \left( i \int_0^{1/T} A_4(\vec{x}, t) dt \right) \xrightarrow{|\vec{x}| \rightarrow \infty} \mathcal{P}_\infty \quad (1)$$

can take arbitrary fixed values  $\mathcal{P}_\infty \notin Z(N_c)$  at spatial infinity. Each single KvBLL (anti)caloron consists of  $N_c$  monopole constituents localized at positions where the

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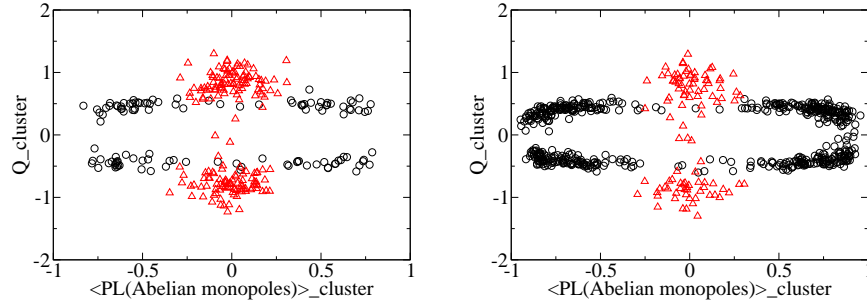
Polyakov loop  $P(\vec{x})$  has degenerated eigenvalues. For  $SU(2)$  this means that  $L(\vec{x}) = \frac{1}{2}\text{tr}P(\vec{x})$  takes opposite values  $\pm 1$  at the positions of the two monopoles. The profile of the Polyakov loop field inside a KvBLL solution is the most significant feature of the new calorons irrespective whether the constituents or 'instanton quarks' are separated or not. If the constituents are far from each other the caloron dissociates into  $N_c$  static non-Abelian monopoles. The topological charge of either lump then depends on the eigenvalue differences of  $\mathcal{P}_\infty$ . The zero modes of the Dirac operator are localized only at one of the constituents. When the fermionic boundary condition is smoothly changed the zero mode jumps from one constituent to another [12] if these are separated.

First we used the cooling method applying it to pure  $SU(2)$  and  $SU(3)$  lattice Monte Carlo gauge fields. We demonstrated that the lumps of topological charge observed in the plateau configurations have to be interpreted in terms of KvBLL calorons [13, 14]. Closer to the deconfinement transition or for a smaller aspect ratio we found an increasing frequency of dissociated monopole pairs in  $SU(2)$ .

More recently we have studied Monte Carlo lattice fields with the 4d smearing method at different temperatures [15]. We found many clusters of topological charge and classified them with respect to their Abelian monopole content (in the maximally Abelian gauge). Two limiting cases suggest an interpretation in terms of KvBLL constituents or calorons: (i) clusters containing a monopole loop winding around the lattice in the time direction, taken as candidates for a single constituent; (ii) clusters containing a closed monopole loop, taken as candidates for undissociated calorons. For these cases we have estimated the topological charge of the cluster,  $Q_{\text{cluster}}$ , and the Polyakov loop averaged over the positions of time-like Abelian monopoles,  $\langle PL \rangle_{\text{cluster}}$ . In the confinement phase - *i.e.* for maximally non-trivial holonomy - we would expect half-integer topological charges for isolated monopoles and integer charge for full calorons. The averaged Polyakov loop should be close to  $\pm 1$  for isolated monopoles and near zero for calorons according to the "dipole" profile of the Polyakov loop inside the KvBLL caloron. What is really observed is seen in the scatter plots in the  $(Q_{\text{cluster}}, \langle PL \rangle_{\text{cluster}})$ -plane of Fig. 1. Each entry corresponds to one of the selected cluster candidates, and the scatter plot is clustering into classes with the expected signatures. In the deconfined phase (not shown), for holonomies closer to the trivial one we would expect to find disbalanced constituents, one with small action and a complementary one with large action. Apart from few full calorons accounting for the topological charge of the configurations, we found many "single-constituent" clusters with static Abelian monopole loops but small topological charge, whereas constituents with topological charges close to  $\pm 1$  were completely missing. We conclude that the model picture of KvBLL calorons may fail in the deconfinement phase.

One can also study the topological content without cooling or smearing techniques by applying purely fermionic methods to equilibrium fields. Such investigations have also provided indications for the presence of KvBLL monopole constituents [16, 17].

What are the consequences if HS calorons are replaced by KvBLL ones as building blocks in a random caloron gas model at finite  $T$ ? Such a model, so far realized in the  $SU(2)$  case [18], requires to start the superposition in the so-called algebraic gauge for which  $A_4(x)$  decreases sufficiently fast. A non-periodic gauge transformation is applied in order to render the gauge field periodic. This restricts us to superpositions of calorons



**FIGURE 1.** Scatter plots of topological charge versus averaged Polyakov loop for topological clusters of lattice fields produced with the Wilson action at  $\beta = 2.3, 2.4$  and lattice size  $24^3 \times 6$  (confinement). Triangles (circles) denote full caloron (isolated static monopole) candidates.

**TABLE 1.** Model parameters  $n(T)$ ,  $\omega(T)$ ,  $\bar{\rho}(T)$ , the lattice grid size  $N_s \times N_\tau$ , and the number of generated configurations  $\#$  for selected temperature values  $T/T_c$ . Furthermore, the measured average Polyakov loop  $\langle |L| \rangle$  (together with the input value  $\cos(2\pi\omega)$ ) and the action surplus factor  $\gamma = \frac{S_{tot}}{N_{caloron} \cdot S_{inst}}$  are given.

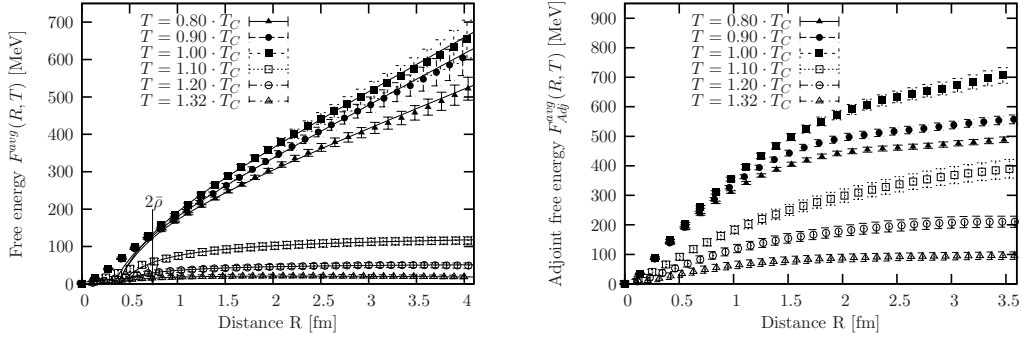
$T/T_c$	$N_s^3 \times N_\tau$	$n^\frac{1}{4}$ [MeV]	$4\omega$	$\bar{\rho}$ [fm]	$\#$	$\cos(2\pi\omega)$	$\langle  L  \rangle$	$\gamma$
0.80	$32^3 \times 10$	198	1.00	0.37	777	0.00	0.13(1)	1.61(1)
1.00	$32^3 \times 8$	198	1.00	0.37	526	0.00	0.14(1)	1.69(1)
1.20	$32^3 \times 8$	174	0.51	0.31	160	0.70	0.59(1)	1.18(1)

with identical holonomy. The positions of the calorons are chosen randomly, the sizes  $\rho$  (*i.e.* the distances  $d = \pi\rho^2/\beta$  between the constituents) are sampled for  $T > T_c$  according to [19]

$$D(\rho, T) = A(T) \cdot \rho^{b-5} \cdot \exp\left(-\frac{4}{3}(\pi T \rho)^2\right), \quad b = 11N_c/3 = 22/3. \quad (2)$$

For  $T < T_c$  temperature independence is postulated but keeping the suppression at  $T_c$  fixed (see [20, 21]). For a statistically uncorrelated caloron gas the actual density  $n(T)$  can be inferred from lattice computations of the topological susceptibility. The average size was fixed by comparison between model and lattice results for the spatial string tension in units of the critical temperature which then turned out to be  $T_c \simeq 178\text{MeV}$ . The holonomy  $\mathcal{P}_\infty \equiv \exp(2\pi i \omega \tau_3)$  was identified with the (renormalized) Polyakov loop. More details and references can be found in [18]. For some parameter sets see Table 1.

On a lattice grid we have computed spatial Wilson loops as well as Polyakov loop correlators, both within fundamental and adjoint representations. The spatial string tension is seen to drop at  $T_c$ , because the mechanism responsible for the observed rise are not the monopoles that are part of the calorons and suppressed at  $T > T_c$ . This problem corresponds to our observation, in the smeared configurations reported above, of many monopoles with low accompanying topological charge. The results for the free energy of a static quark-antiquark pair are shown in Fig. 2. We can follow a linear rise for distances up to  $O(4\text{fm})$  in the confinement phase, whereas it becomes screened above  $T_c$ . The free



**FIGURE 2.** Colour averaged free energy versus distance  $R$  at various temperatures for the fundamental (left) and adjoint (right) representations.

energy of adjoint charge pairs is screened in both phases.

We conclude that a semiclassical model for finite-temperature  $SU(2)$  fields should start from calorons with generic holonomy. Such a model turns out to describe confinement with parameters which are rather close to standard instanton model assumptions. Keeping all parameters fixed, merely changing the holonomy from  $\omega = 1/4$  to  $\omega = 0$  or  $1/2$ , removes the linear rise completely [18]. This underscores the rôle of non-trivial holonomy and the corresponding long-range nature of the caloron fields.

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